1st Semester Examination, 2020

Time : 3 hours

Full Marks : 60

Answer any **one** Group as per your Syllabus.

Answer from all the sections as per direction.

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP—A
(MODEL SYLLABUS)
(MATHEMATICAL PHYSICS-I)

SECTION–A

1. Answer *all* questions : 1×8

(*a*) $y = x$ is a straight line of slope ——— .

(*b*) $\lim_{x \to 0} \frac{\sin x}{x} =$ ——— .

(*c*) Order of the differential equation $\frac{d^2 y}{dx^2} + 6y = 2x$, is ——— .

(*d*) If $U = e^x \cos y$, then $\frac{\partial u}{\partial x} =$ ——— .

(*e*) Angle between the two meters $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{B} = 5\hat{i} + 2\hat{j} + 4\hat{k}$ is ——— .

(*f*) For any constant a and dirac delta function $\delta(x)$ $\frac{\delta(-x)}{a} =$ ——— .

(*g*) Divergence of a solenoidal vector is ——— .

(*h*) $\iint_S \vec{F} \cdot \hat{n} ds =$ ——— .

SECTION–B

2. Answer any *eight* of the following : 1.5 × 8

(*a*) Plot the graph $y = x^2$.

(*b*) Find $\lim\limits_{x \to 0} \frac{\left(\sqrt{1-x}\right)-1}{x}$.

(*c*) Find the general solution of the differential equation $ax\frac{dy}{dx} = by$.

(*d*) Show that the function e^{ax} and e^{-ax} and linearly independent.

(*e*) Check the continuity of the function $f(x, y) = x^2 + 2y$, at (1, 2).

(*f*) Solve, $ydx - xdy = xy^3dy$.

(*g*) Justify the statement – 'If three vectors are co-planar, then the value of the scalar tripple product is zero'.

(*h*) Draw and define cylindrical co-ordinates.

(*i*) Show that grad $(\varphi + \psi)$ = grad φ + grad ψ.

(*j*) Show that $\int_C \vec{r} \cdot d\vec{r} = 0$.

SECTION–C

3. Answer any *eight* of the following : 2 × 8

(*a*) Find $\lim\limits_{x \to 0} \frac{x^2 + 8x}{x}$.

(*b*) Find $\frac{dy}{dx}$, if $x = a\,(t + \sin t)$, $y = a \cos t$.

(*c*) With a suitable example define a homogeneous differential equation of degree *n*.

(*d*) Solve the differential equation $x\frac{dy}{dx} + y = x^3 + x$.

(*e*) What is wronskian ? What is its application ?

(*f*) Solve the differential equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^{3x}$.

(*g*) Evaluate $\lim\limits_{\substack{x\to0\\y\to0}} \frac{xy}{x^2+y^2}$.

(*h*) What is the geometrical interpretation of gradient of a function.

(*i*) For dirac delta function show that

$$f(x)\ \delta(n-a)=f(a)\ \delta(x-a)$$

(*j*) Using Green's theorem show that area of a plane region

$$A=\frac{1}{2}\oint_C (xdy-ydx)$$

SECTION–D

Answer *all* questions :

4. Draw the graph $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and check the continuity and differentiability of the function. 6

Or

Solve the differential equation $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+3x=\sin t$. 6

5. If $U=\frac{y}{z}+\frac{z}{x}$, then find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$. 6

Or

For unit vectors $\hat{i}, \hat{j}$ & $\hat{k}$ and for any vector $\vec{A}$ find

$$\hat{i}\times\left(\vec{A}\times\hat{i}\right)+\hat{j}\times\left(\vec{A}\times\hat{j}\right)+\hat{k}\times\left(\vec{A}\times\hat{k}\right)$$ 6

6. Find the expression for velocity in spherical polar coordinates. 6

Or

Discuss the properties of Dirac delta function. 6

7. Justify the statement for two scalar function f and g, $\nabla f \times \nabla g$ is solenoidal. 6

Or

State and prove Stoke's theorem. 6

GROUP—B
(OLD SYLLABUS)
(MATHEMATICAL PHYSICS-I)

SECTION–A

1. Answer *all* questions : 2×6

(*a*) Define vector triple product.

(*b*) Find the gradient of scalar function $\phi(xy) = x^2 - y^2$.

(*c*) Explain Lagrange method of undetermined multipliers.

(*d*) Prove that $\delta(-x) = \delta(x)$.

(*e*) Write relation between unit vectors $(\hat{r}, \hat{\theta})$ and $(\hat{i}, \hat{j})$.

(*f*) State Green's theorem.

SECTION–B

Answer *all* questions : 12×4

2. (*a*) Explain scalar triple product with its physical significance and features. 10

(*b*) Distinguish between scalar field and vector field. 2

Or

(*a*) Explain divergence of a vector field and its physical significance with examples. 10

(*b*) Calculate the divergence of the vector

$$\vec{A} = xy\hat{i} + yz\hat{j} + zx\hat{k} \cdot$$ 2

3. (*a*) Derive an expression for partial differentiation of vectors. 10

(*b*) Solve the differential equation $\left(1+x^2\right)dy-\left(1+y^2\right)dx=0$. 2

Or

(*a*) Derive relation of Delta function with the step function. 6

(*b*) Prove that $x\cdot\delta(x)=0$. 3

(c) Prove that $f(x)\cdot\delta(x-a)=f(a)\cdot\delta(x-a)$. 3

4. Find the divergence and curl in terms of orthogonal curvilinear coordinates. 6+6

Or

Derive an expression for velocity and acceleration in cylindrical and spherical polar coordinates in three dimension. 6+6

5. State and prove Gauss divergence theorem with its physical significance. 12

Or

Explain line integral, surface integral and volume integral of a vector function. 4+4+4